

# Dipole emission near a planar multilayer stack

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## Abstract

The `planar` package solves the electromagnetic problem of dipole emission near a planar multilayer stack. It comprises two sets of functions; i) to compute the effective Fresnel reflection coefficient of a multilayer structure; ii) to evaluate the modified dipolar field as an integral over plane waves reflected at the interface.

## 1 Fresnel coefficients

The functions `recursive.fresnel` and `multilayer` both compute the Fresnel coefficients for a multilayer stack, using two different methods (recursive application of Fresnel coefficients for a layer; and transfer matrix, respectively).

### 1.1 Multilayer optics

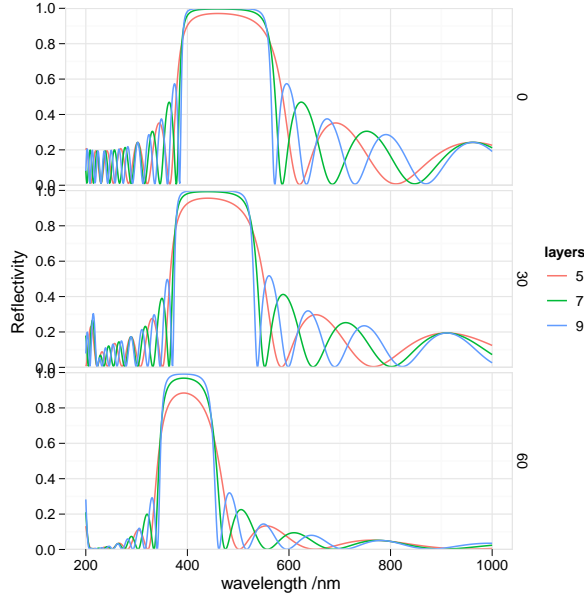


Figure 1: `demo(bragg_stack)` from the planar package. Reflectivity of a Bragg stack with varying number of layers. Reproducing Fig. 6.6, p. 188 of Mac Leod's Thin Film Optical Filters the structure is a stack of  $\lambda/4$  layers of indices  $n_H$  and  $n_L$  on a glass substrate with increasing number of layers, the reflectivity stop-band becomes stronger.

## 1.2 Kretschmann configuration – planar surface plasmon-polaritons

First, we look at the reflectivity of a thin metal film excited in the Kretschmann configuration.

In the same configuration, SPPs may be excited for a wide range of frequencies. The dispersion of the surface mode may be observed as a high reflectivity trace when plotted as a function of incident in-plane wavevector and energy.

Free-space radiation cannot directly couple to SPP modes due to a momentum mismatch. Using evanescent illumination, in-plane wavevectors of arbitrarily large value may be obtained and allow the mapping of the coupled-SPPs dispersion in a symmetric configuration.

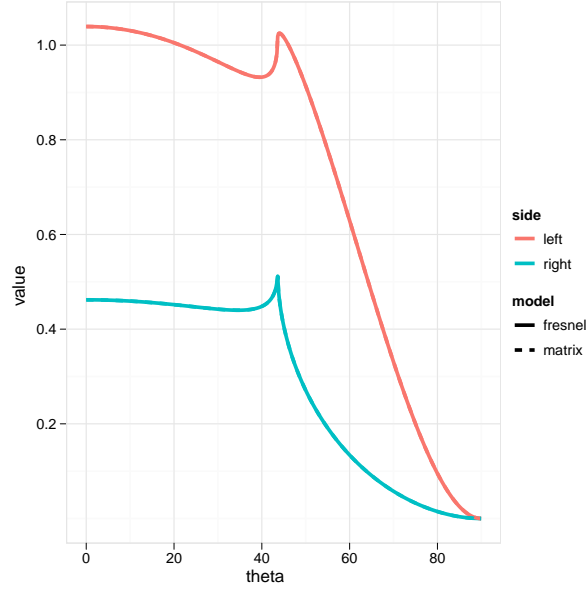


Figure 2: `demo(field_enhancement)` from the planar package. Comparison of the calculation of near field enhancement outside of a thin metal film with Fresnel reflection and transmission coefficients.

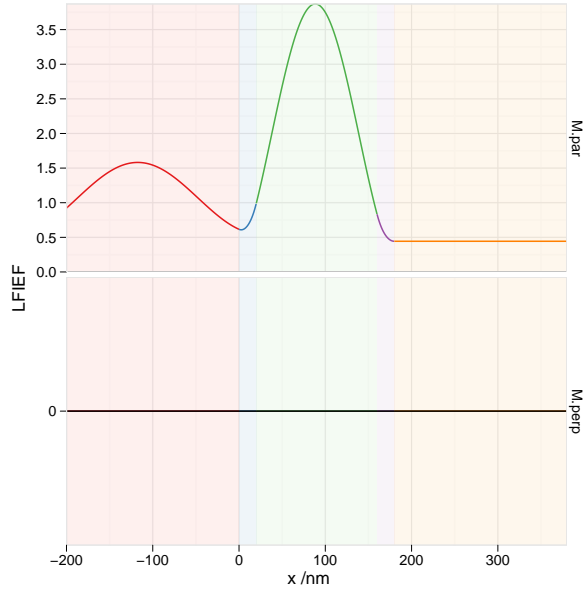


Figure 3: `demo(LFIEF_distance)` from the planar package. Local field enhancement factors for a dipole near or inside a multilayer. Note that the field and its derivative are continuous across all interfaces.

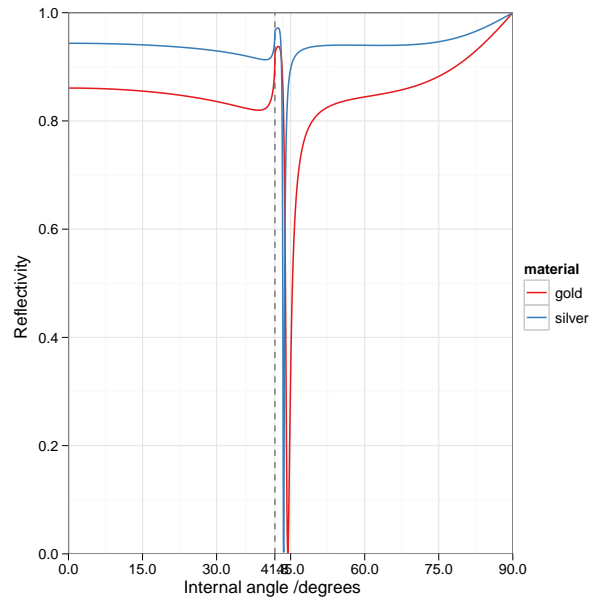


Figure 4: Reflectivity of a thin metal film, 50 nm thick, sandwiched between glass ( $n = 1.5$ ) and air. The SPP is excited at the metal/air interface. By changing the incident angle, the normalised in-plane wavevector  $q$  varies from 0 (normal incidence) to 1 (grazing internal angle).

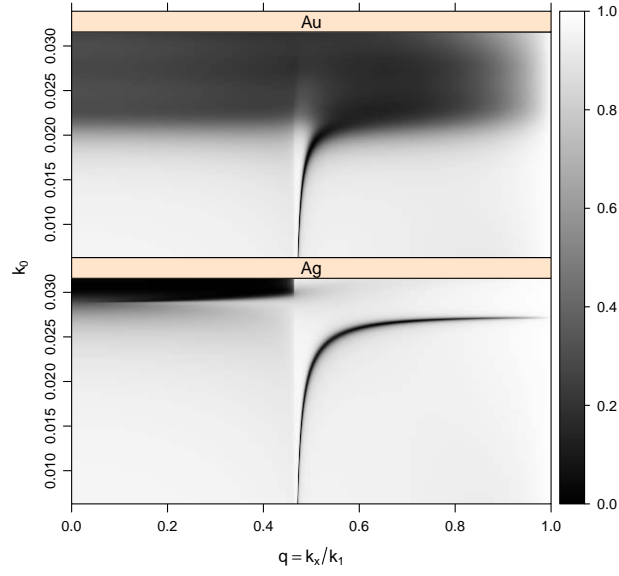


Figure 5: Reflectivity of a thin metal film, 50 nm thick, sandwiched between semi-infinite glass ( $n = 1.5$ ) and air. The dispersion of the SPP mode appears as a dark curve following the equation  $k_{\text{spp}} = k_0 \sqrt{\frac{\epsilon_{\text{metal}} \epsilon_{\text{air}}}{\epsilon_{\text{metal}} + \epsilon_{\text{air}}}}$

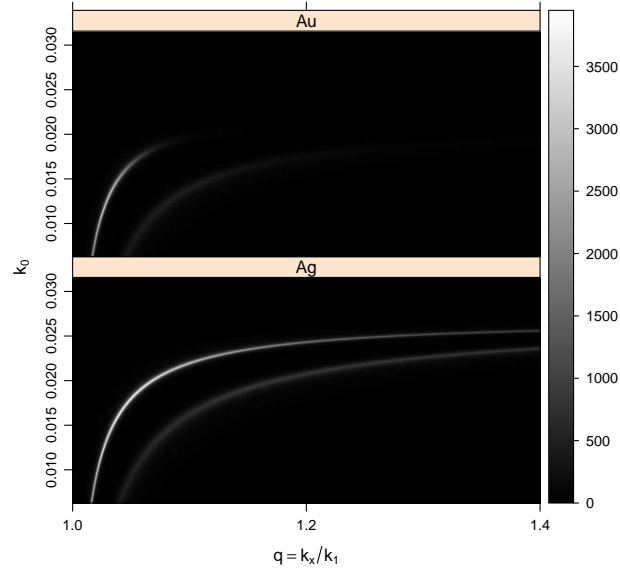


Figure 6: Reflectivity of a thin metal film, 50 nm thick, sandwiched between semi-infinite glass ( $n = 1.5$ ) on either side. Coupled SPPs are excited when the normalized in-plane wavevector  $q$  is greater than 1. Note that values of  $|r|^2 > 1$  are not unphysical, as no power is transferred by evanescent waves.

## 2 Decay rates

From [RE09] (p. 571), and [NH06] (pp. 335–360), the total decay rate for a dipole perpendicular to the interface is

$$M_{\text{tot}}^{\perp} = 1 + \frac{3}{2} \int_0^{\infty} \Re \left\{ \frac{q^3}{\sqrt{1-q^2}} r^p(q) \exp \left( 2ik_1 d \sqrt{1-q^2} \right) \right\} dq \quad (1)$$

The integrand diverges as  $q \rightarrow 1$ , it is therefore advantageous to perform the substitution  $u := \sqrt{1-q^2}$ . In order to maintain a real path of integration, the integral is first split into a radiative region ( $0 \leq q \leq 1$ ,  $u := \sqrt{1-q^2} \geq 0$ ), and an evanescent region ( $1 \leq q \leq \infty$ ,  $-iu := \sqrt{q^2-1} \geq 0$ ). After some algebraic manipulation, we obtain,

$$M_{\text{tot}}^{\perp} = 1 + \frac{3}{2} (I_1 + I_2) \quad (2)$$

where

$$\begin{aligned} I_1 + I_2 = & \int_0^1 [1 - u^2] \cdot \Re \left\{ r^p(\sqrt{1-u^2}) \exp(2idk_1 u) \right\} du \\ & + \int_0^{\infty} [1 + u^2] \cdot \exp(-2dk_1 u) \cdot \Im \left\{ r^p(\sqrt{1+u^2}) \right\} du \end{aligned} \quad (3)$$

Similarly, for the parallel dipole

$$M_{\text{tot}}^{\parallel} = 1 + \frac{3}{4} \int_0^{\infty} \Re \left\{ \left[ \frac{r^s(q)}{\sqrt{1-q^2}} - r^p(q) \sqrt{1-q^2} \right] \cdot q \cdot \exp \left( 2ik_1 d \sqrt{1-q^2} \right) \right\} dq \quad (4)$$

which can be rewritten as,

$$M_{\text{tot}}^{\parallel} = 1 + \frac{3}{4} (I_1^{\parallel} + I_2^{\parallel}) \quad (5)$$

where

$$\begin{aligned} I_1^{\parallel} + I_2^{\parallel} = & \int_0^1 \Re \left\{ \left[ r^s(\sqrt{1-u^2}) - u^2 \cdot r^p(\sqrt{1-u^2}) \right] \exp(2idk_1 u) \right\} du \\ & + \int_0^{\infty} \exp(-2dk_1 u) \cdot \Im \left\{ r^s(\sqrt{1+u^2}) + u^2 \cdot r^p(\sqrt{1+u^2}) \right\} du \end{aligned} \quad (6)$$

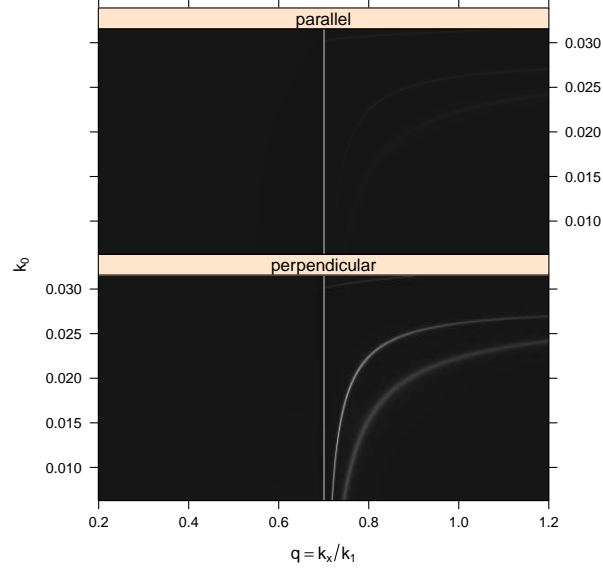


Figure 7: Integrand in the resonance region of the total decay rate enhancement factor  $M_{\text{tot}}$  for a dipole situated 5 nm above a metal interface.

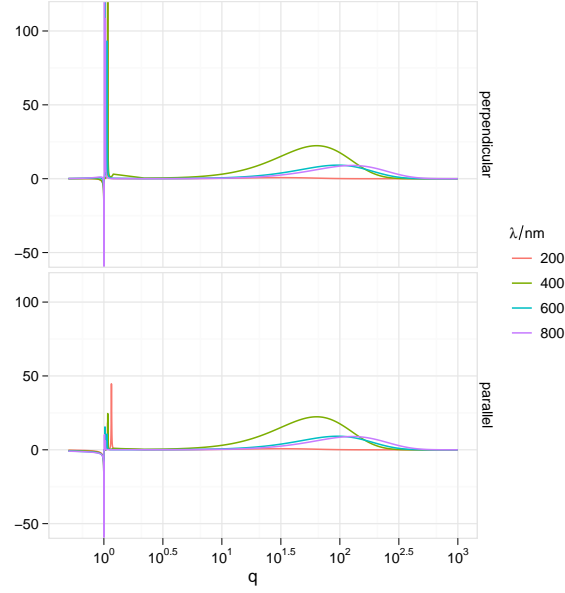


Figure 8: Integrand of the total decay rate enhancement factor  $M_{\text{tot}}$  for a dipole situated 5 nm above a metal interface, for several emission wavelengths.



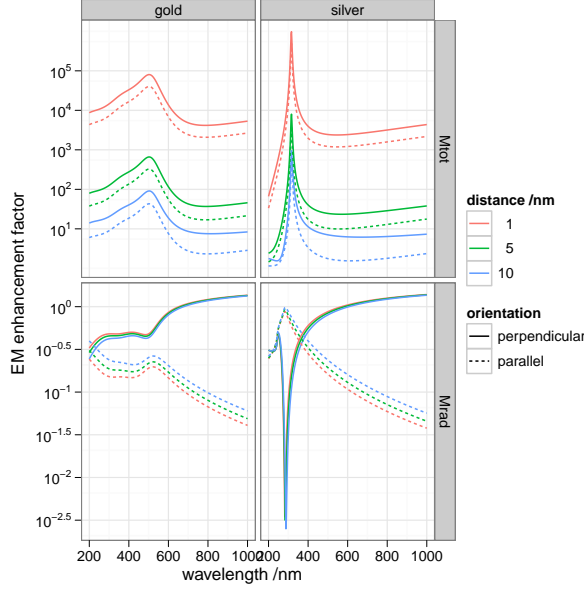


Figure 9: Total and radiative decay rate enhancements for a dipole near a metal interface. Reproducing Fig. 6.1, p. 304 from Principles of Surface-Enhanced Raman Spectroscopy. A dipole is placed near a semi-infinite air/metal interface with orientation either parallel or perpendicular to the interface the total decay rates peak at the wavelength of excitation of planar SPPs  $\epsilon = -1$  at the interface (loss channel). The radiative decay rate in the upper medium has a trough at the wavelength where  $\epsilon = 0$  ( $Dn = 0$ , by continuity  $En = 0$ ).

## 2.1 Angular pattern of dipole emission

By virtue of reciprocity, the local field intensity enhancement factor also represent the probability of emission of a dipole in a particular direction.

## References

- [NH06] Lukas Novotny and B. Hecht. *Principles of Nano-Optics*. Cambridge Univ Pr, January 2006.
- [RE09] Eric Le Ru and Pablo Etchegoin. *Principles of Surface-Enhanced Raman Spectroscopy*. Elsevier, 2009.

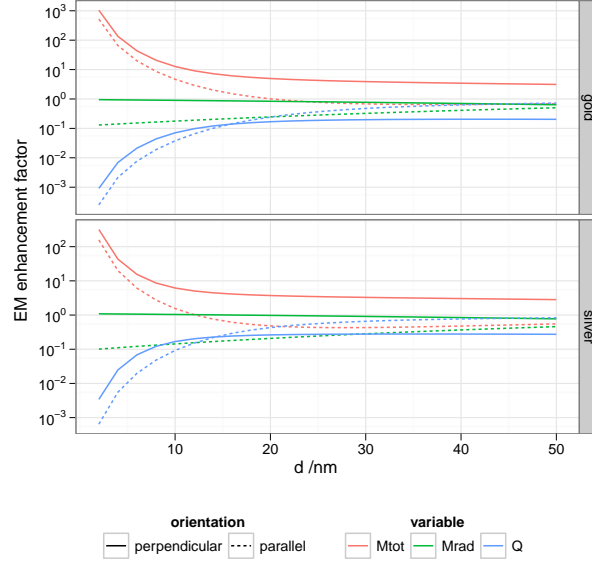


Figure 10: Integrated decay rates and efficiency for a dipole near a semi-infinite air/metal interface for gold and silver, varying the wavelength and the dipole-interface distance.

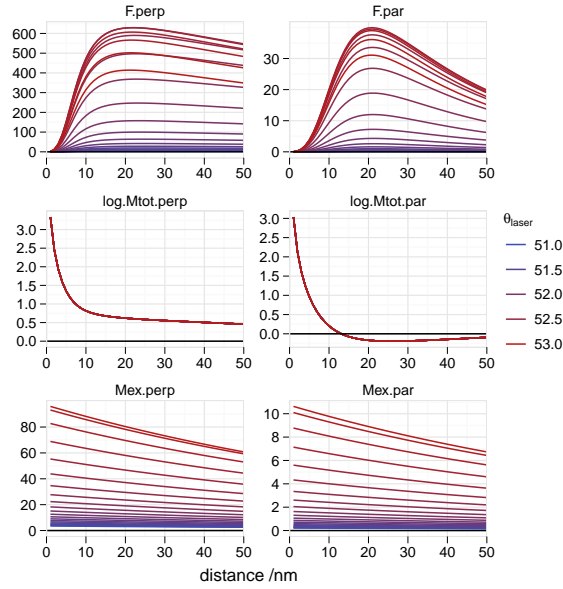


Figure 11: Fluorescence enhancement vs distance in the Kretschmann configuration.

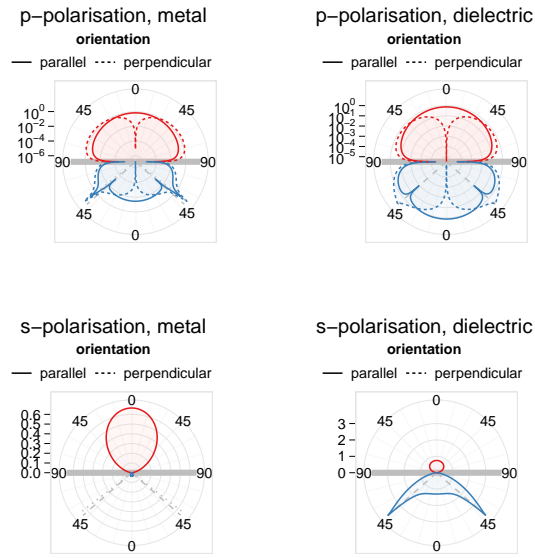


Figure 12: Radiation pattern of a dipole near a dielectric/(metal)/dielectric interface parallel and perpendicular orientations, p- and s- polarisations.