

# Package ‘pmlsp’

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**Title** Partial Maximum Likelihood Estimation of Spatial Probit Models

**Version** 1.0.1

**Description** Estimate spatial autoregressive nonlinear probit models with and without autoregressive disturbances using partial maximum likelihood estimation. Estimation and inference regarding marginal effects is also possible. For more details see Bille and Leorato (2020) <[doi:10.1080/07474938.2019.1682314](https://doi.org/10.1080/07474938.2019.1682314)>.

**License** GPL (>= 3)

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**Imports** abind, Matrix, matrixcalc, maxLik, methods, minqa, mvtnorm, numDeriv, qrng, spatialreg, spdep, stats, utils

**Depends** R (>= 3.5)

**LazyData** true

**URL** <https://github.com/d-spinelli/pmlsp>

**BugReports** <https://github.com/d-spinelli/pmlsp/issues>

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## Contents

|                           |   |
|---------------------------|---|
| ape . . . . .             | 2 |
| coef.pmlsprobit . . . . . | 3 |

|                                    |    |
|------------------------------------|----|
| eurostat . . . . .                 | 4  |
| pmlsbp . . . . .                   | 4  |
| predict.pmlsprobit . . . . .       | 8  |
| print.summary.pmlsprobit . . . . . | 9  |
| summary.pmlsprobit . . . . .       | 10 |
| vcov.pmlsprobit . . . . .          | 11 |

|              |           |
|--------------|-----------|
| <b>Index</b> | <b>12</b> |
|--------------|-----------|

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ape *Average Marginal Effects for Spatial Probit Models*

---

### Description

Computes average direct, indirect, and total marginal effects from an object of class "pmlsprobit" using numerical derivatives.

### Usage

```
ape(mod, variables = NULL)
```

### Arguments

|           |   |
|-----------|---|
| mod       | An object of class "pmlsprobit".  |
| variables | Character vector specifying the names of covariates for which marginal effects are computed. This option should be used only when the model contains non-linear terms in the specification $X\beta$ . |

### Value

A named list with three elements corresponding to the average direct, indirect, and total marginal effects for each covariate.

### See Also

Partial maximum likelihood estimation is implemented in [pmlsbp](#).

### Examples

```
data(oldcol, package = "spdep")
dat <- COL.OLD
dat$y <- as.numeric(dat$CRIME > 35)

listw <- spdep::nb2listw(COL.nb, style = "W")

set.seed(857489)
mod <- pmlsbp(
  y ~ HOVAL + INC, data = dat, W = listw,
  model = "SAR", grouping = 7,
```

```
    mvtnorm_control = list(  
      M = 25e2, sim_type = "mc",  
      tol = .Machine$double.eps, fast = FALSE  
    ),  
    na.action = na.omit, spectral = TRUE,  
    iterlim = 1e5  
  )  
  
ape(mod)
```

---

|                 |   |
|-----------------|---|
| coef.pmlsprobit | <i>Coefficient Estimates for Partial Maximum Likelihood Spatial Probit Models</i> |
|-----------------|---|

---

### Description

Extracts the estimated regression coefficients from an object of class "pmlsprobit".

### Usage

```
## S3 method for class 'pmlsprobit'  
coef(object, ...)
```

### Arguments

|        |  |
|--------|--|
| object | An object of class "pmlsprobit".         |
| ...    | Additional arguments (currently unused). |

### Value

A named numeric vector containing the coefficient estimates.

### See Also

Partial maximum likelihood estimation is implemented in [pmlsbp](#).

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eurostat

*Eurostat employment data*

---

### Description

The eurostat data frame has 312 row and 10 variables. Unit of analysis: 312 regions in Europe. Additionally, the eurostat.nb includes the neighbour list based on a consider a k-nearest neighbour approach with k = 11

### Usage

```
data(eurostat)
```

### Format

A data frame with 312 rows and 10 variables:

nuts\_id Region identifier

country\_id Country identifier

employment Employment rate in percentage

gdp GDP per capita

isced\_02 percentage of residents aged 25-64 with secondary school achievement or lower

isced\_34 percentage of residents aged 25-64 with high school achievement or lower

isced\_58 percentage of residents aged 25-64 with bachelor degree or higher

long Longitude

lat Latitude

### Source

<https://ec.europa.eu/eurostat/data/database>

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pmlsbp

*Fitting Spatial Probit Models by Partial Maximum Likelihood*

---

### Description

Fits spatial autoregressive probit models using partial maximum likelihood as proposed in Bille and Leorato (2020) doi:10.1080/07474938.2019.1682314. Both Spatial Autoregressive (SAR) probit models and Spatial Autoregressive Probit Models with Autoregressive Disturbances (SARAR) are supported.

**Usage**

```

pmlsbp(
  formula, data, model = "SAR", grouping = 2,
  W = NULL,
  zero.policy = spatialreg::get.ZeroPolicyOption(),
  M = NULL, formula_xlag = NULL, W2 = NULL,
  method_inv = "solve",
  start = NULL, subset = NULL, na.action = na.fail,
  qu = Inf, iterlim = 1000,
  mvtnorm_control = list(M = 1e4, sim_type = "qmc",
                        tol = .Machine$double.eps, fast = FALSE),
  finalHessian = ifelse(method == "bhhh", method, TRUE),
  method = "bhhh", print.level = 2,
  vce.type = "asy",
  Conley = list(coords = NULL, LM = 2),
  nBoot = 1e3, spectral = FALSE,
  verbose = TRUE ,
  tol.solve = .Machine$double.eps,
  version = 0, ...
)

## S3 method for class 'pmlsprobit'
print(x, digits = max(3, getOption("digits") - 3), ...)

```

**Arguments**

|              |  |
|--------------|--|
| formula      | An object of class "formula" specifying the model.   |
| data         | An optional data frame containing the variables in the model.  |
| model        | A character string specifying the model type; either "SAR" or "SARAR".   |
| grouping     | An integer defining the number of observations to include in the tuples used for estimation.                       |
| W            | A spatial weights object of class "listw", for example created by <a href="#">nb2listw</a> .                       |
| zero.policy  | Logical; if TRUE, assigns zero to lagged values for observations without neighbors.                                |
| M            | A spatial weights object of class "listw" used for the disturbance process; relevant only if model = "SARAR".      |
| formula_xlag | An optional "formula" specifying covariates to be spatially lagged.  |
| W2           | A spatial weights object used to construct spatially lagged covariates; relevant only if formula_xlag is not NULL. |
| method_inv   | Character string specifying the method used to invert $I - \rho W$ ; one of "solve", "chol", or "fast".            |
| start        | Numeric vector of starting values.   |
| subset       | Optional vector specifying a subset of observations.   |
| na.action    | A function specifying how missing values are handled.  |

|                 |  |
|-----------------|--|
| qu              | Integer used only if <code>method_inv = "fast"</code> .  |
| iterlim         | Integer specifying the maximum number of iterations.   |
| mvtnorm_control | A list of control parameters for multivariate normal probability calculations; see <a href="#">lpmvnorm</a> .  |
| finalHessian    | Logical or character specifying how the final Hessian matrix is computed; see <a href="#">maxLik</a> .   |
| method          | Maximization method passed to <a href="#">maxLik</a> .   |
| print.level     | Integer controlling the amount of diagnostic output.   |
| vce.type        | Character specifying the variance–covariance estimator; one of "asy", "bootstrap", or "mConley".   |
| Conley          | A list specifying options for the modified Conley estimator.   |
| nBoot           | Integer specifying the number of bootstrap replications.   |
| spectral        | Logical; if TRUE, uses spectral normalization of $W$ .   |
| verbose         | Logical; if TRUE, diagnostic messages are printed during model estimation. The default is TRUE. Setting <code>verbose = FALSE</code> suppresses intermediate output. |
| tol.solve       | Numeric tolerance used when inverting matrices.  |
| version         | Integer controlling the computation of bivariate normal integrals when grouping = 2.   |
| ...             | Additional arguments passed to <a href="#">maxLik</a> .  |
| x               | a "pmlsprobit" object  |
| digits          | integer, used for number formatting the decimal digits   |

## Details

The SARAR model is defined as

$$y^* = \rho W y + X\beta + W_2 Z\gamma + u$$

with

$$u = \lambda M u + \epsilon,$$

where  $\epsilon \sim N(0, \sigma^2)$  and the observed binary outcome is  $y = 1(y^* > 0)$ . The SAR model is obtained by setting  $\lambda = 0$ .

## Value

An object of class "pmlsprobit", which is a list containing:

- beta Named vector of regression coefficients.
- call The matched function call.
- code Convergence code from [maxLik](#).
- estimate Named vector of parameter estimates.
- f List of matrices used for prediction.

maximum Value of the partial log-likelihood.  
 message Convergence message from maxLik.  
 model Model frame used in estimation.  
 model.type Either "SAR" or "SARAR".  
 rho Estimated spatial autoregressive parameter.  
 slx Logical indicating the presence of spatially lagged covariates.  
 start Starting values used in estimation.  
 terms Terms object.  
 vcov Variance–covariance matrix of the estimates.  
 W Spatial weights matrix  $W$ .  
 lambda Estimated disturbance parameter (SARAR only).  
 M Spatial weights matrix  $M$  (SARAR only).  
 W2 Spatial weights matrix  $W_2$  (if slx = TRUE).  
 No return value

## Functions

- `print(pmlsprobit)`: prints the call and the coefficient estimates of a "pmlsprobit" object

## References

Bille', A. G., & Leorato, S. (2020). Partial ML estimation for spatial autoregressive nonlinear probit models with autoregressive disturbances. *Econometric Reviews*, 39(5), 437–475. doi:10.1080/07474938.2019.1682314

## See Also

[pmlsbp](#), [maxLik](#)

## Examples

```

data(oldcol, package = "spdep")
dat <- COL.OLD
dat$y <- as.numeric(dat$CRIME > 35)

listw <- spdep::nb2listw(COL.nb, style = "W")

set.seed(857489)
mod <- pmlsbp(
  y ~ HOVAL + INC, data = dat, W = listw,
  model = "SAR", grouping = 7,
  na.action = na.omit, spectral = TRUE,
  iterlim = 1e5
)

```

---

predict.pmlsprobit      *Predicted Values and Marginal Effects for Spatial Probit Models*

---

### Description

Computes predicted probabilities or average marginal effects from an object of class "pmlsprobit".

### Usage

```
## S3 method for class 'pmlsprobit'
predict(
  object,
  type = "prob",
  newdata = NULL,
  variables = NULL,
  delta.method = FALSE,
  ...
)
```

### Arguments

|              |   |
|--------------|---|
| object       | An object of class "pmlsprobit".  |
| type         | Character string specifying the type of prediction. Use "prob" for predicted probabilities $P(Y_i = 1)$ , or "me" for direct, indirect, and total average marginal effects. |
| newdata      | Optional data frame containing variables used for prediction. If omitted, the fitted values are used.   |
| variables    | Character vector of covariate names for which marginal effects are computed using numerical derivatives. Only used if type = "me".  |
| delta.method | Logical; if TRUE, standard errors of the predictions are computed. Only used if type = "me".  |
| ...          | Additional arguments (currently unused).  |

### Value

If type = "prob", a numeric vector of predicted probabilities. If type = "me", a list of matrices containing individual direct, indirect, and total marginal effects for each covariate.

### See Also

Partial maximum likelihood estimation is implemented in [pmlsbp](#).

### Examples

```
data(oldcol, package = "spdep")
dat <- COL.OLD
dat$y <- as.numeric(dat$CRIME > 35)

listw <- spdep::nb2listw(COL.nb, style = "W")

set.seed(857489)
mod <- pmlsbp(
  y ~ HOVAL + INC, data = dat, W = listw,
  model = "SAR", grouping = 7,
  mvtnorm_control = list(
    M = 25e2, sim_type = "mc",
    tol = .Machine$double.eps, fast = FALSE
  ),
  na.action = na.omit, spectral = TRUE,
  iterlim = 1e5
)

prob <- predict(mod)
head(prob)
```

---

`print.summary.pmlsprobit`

*Print method for summary.pmlsprobit objects*

---

### Description

S3 method to print objects of class `summary.pmlsprobit`.

### Usage

```
## S3 method for class 'summary.pmlsprobit'
print(x, digits = max(3L, getOption("digits") - 3L), ...)
```

### Arguments

|                     |   |
|---------------------|---|
| <code>x</code>      | An object of class <code>summary.pmlsprobit</code> .    |
| <code>digits</code> | Number of significant digits to print for coefficients. |
| <code>...</code>    | Additional arguments (currently ignored).               |

### Value

No return value

---

summary.pmlsprobit      *Summarizing Partial Maximum Likelihood Spatial Probit Models*

---

### Description

Provides a summary method for objects of class "pmlsprobit".

### Usage

```
## S3 method for class 'pmlsprobit'
summary(object, ...)
```

### Arguments

object            An object of class "pmlsprobit".  
 ...                Additional arguments (currently unused).

### Value

An object of class "summary.pmlsprobit" containing:

- estimate Matrix of estimates, standard errors, z-values, and p-values.
- rho Estimated spatial autoregressive parameter.
- lambda Estimated spatial error parameter.
- model.type Type of spatial probit model fitted.
- loglik Log-likelihood at the optimum.
- iteration Number of iterations performed.
- returnCode Optimizer convergence code.
- returnMessage Message associated with the convergence code.
- vcov Variance-covariance matrix of the parameter estimates.

### See Also

Partial maximum likelihood estimation is implemented in [pmlsbp](#).

### Examples

```
data(oldcol, package = "spdep")
dat <- COL.OLD
dat$y <- as.numeric(dat$CRIME > 35)

listw <- spdep::nb2listw(COL.nb, style = "W")

set.seed(857489)
mod <- pmlsbp(
  y ~ HOVAL + INC, data = dat, W = listw,
```

```
model = "SAR", grouping = 7,  
na.action = na.omit, spectral = TRUE,  
iterlim = 1e5  
)  
summary(mod)
```

---

|                 |  |
|-----------------|--|
| vcov.pmlsprobit | <i>Variance–Covariance Matrix for Partial Maximum Likelihood Spatial Probit Models</i> |
|-----------------|--|

---

### Description

Extracts the variance–covariance matrix from an object of class "pmlsprobit".

### Usage

```
## S3 method for class 'pmlsprobit'  
vcov(object, ...)
```

### Arguments

|        |  |
|--------|--|
| object | An object of class "pmlsprobit".         |
| ...    | Additional arguments (currently unused). |

### Value

A numeric matrix containing the variance–covariance matrix of the parameter estimates.

### See Also

Partial maximum likelihood estimation is implemented in [pmlsbp](#).

# Index

## \* datasets

eurostat, 4

ape, 2

coef.pmlsprobit, 3

eurostat, 4

lpmvnorm, 6

maxLik, 6, 7

nb2listw, 5

pmlsbp, 2, 3, 4, 7, 8, 10, 11

predict.pmlsprobit, 8

print.pmlsprobit (pmlsbp), 4

print.summary.pmlsprobit, 9

summary.pmlsprobit, 10

vcov.pmlsprobit, 11